Removal of supersonic ion singularity in radial Langmuir probe models

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It is well known that a singularity appears when the ions reach the speed of sound in an electropositive plasma. For cold ions, the singularity is at infinity, and so it poses no problem the numerical integration of radial Langmuir probe models. However, for warm ions the singularity typically occurs between the quasi-neutral plasma and the sheath. We have found that we can continuously join the solution at the plasma with the probe thanks to a careful analysis of the mathematical structure of the problem. The technique can be applied to different geometries and to electronegative plasmas as well. For the case of cylindrical Langmuir probes, we have derived potential profiles, ion population profiles and ion current to probe voltage characteristics. These results are used to refine diagnosis techniques by means of Langmuir probes in laboratory plasmas.

In the interest of obtaining the potential profile $\phi(r)$ around a Langmuir probe in electropositive plasmas one should solve **Poisson's equation**. In the case of cylindrical geometry, we have

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\phi}{\mathrm{d}r}\right) = -\frac{e}{\varepsilon_0}\left[n_+(r) - n_e(r)\right]. \qquad (1)$$

The electron density $n_e(r)$ will be described by the Maxwellian distribution function, whereas the ion density $n_+(r)$ depends on the ion motion theory used. When using a radial motion theory**, the thermal motion of the ions introduces an additional term in the energy balance equation** [1], giving

$$\frac{1}{2}m_+v_+^2(r) + e\phi(r) + \frac{\kappa}{\kappa-1}k_BT_+\left(\frac{n_{+(r)}}{n_{e0}}\right)^{\kappa-1} = \frac{\kappa}{\kappa-1}k_BT_+, \qquad (2)$$

where κ is the adiabatic coefficient of the thermal flow. As $v_+(r)$ is inversely related to the ion density through continuity equation, i being the ion current per unit length collected by the probe,

$$i = e2\pi n_+(r)v_+(r), \qquad (3)$$

we get a polynomic equation in $n_+(r)$, with defining parameters r and $\phi(r)$, which should be solved in order to introduce its value into Poisson's equation. We have found that **this polynomial has two positive roots** that coalesce into one for certain values of the problem variables r and ϕ. We further found that one of the roots is valid in the plasma in the limit $x \to \infty$, where the ions are at rest, while the other root is valid in the sheath in the cold ions limit $T_+ \to 0$.

We have proved that the transition between these roots must occur, in the variable space (r, ϕ), in the curve where the two positive roots of the energy balance polynomial coalesce in a sort of bifurcation line, and that the only possible smooth and continuous crossing through the **regular singularity [2] where the ions reach the speed of sound** is tangent to that bifurcation curve. In figure 1 we show an example of solution of the potential profile.

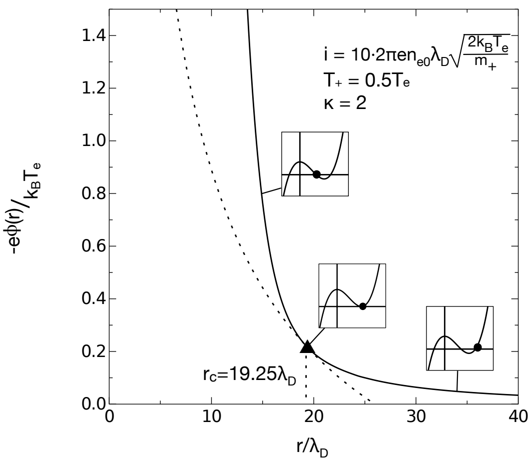

Figure 1: Potential profile and bifurcation curve solution. The small inserts are qualitative plots of the polynomic equation for $n_+(r)$.

This method is **valid for any ion temperature**. We indicate with a triangle the point where the speed of sound is reached. To the plasma or to the sheath we use the right energy balance polynomial root, as we mark with a dot in the inserts in the figure.

2. References

[1] J.I. Fernández Palop *et al* 1996 *J. Phys. D: Appl. Phys* **29** 2831.

[2] H.B. Valentini 1988 *J. Phys. D: Appl. Phys* **21** 311-321