Calculation of electron velocity distribution function under crossed electric and magnetic fields using a propagator method

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A propagator method (PM) to calculate the Boltzmann equation (BE) for the electron velocity distribution function (EVDF) in gas was extended for that under crossed electric and magnetic ($\boldsymbol{E}\times\boldsymbol{B}$) fields. Three-variable velocity space was divided into cells and the number of electrons in each cell was calculated with a three-dimensional memory array. The propagators to operate the intercellular electron transfers due to acceleration and collisional scattering were customized for the cell configuration and the electron acceleration particular to the $\boldsymbol{E}\times\boldsymbol{B}$ fields. Equilibrium EVDFs at some E/N and B/N values were obtained using a numerical relaxation scheme, and electron transport parameters derived from the EVDFs agreed with those obtained by a Monte Carlo (MC) simulation.

1. Introduction

In a PM [1] to solve the BE for the EVDF, velocity space is divided into cells and the number of electrons in each cell is calculated. While the EVDF under a dc $\boldsymbol{E}$ field is a two-variable function for its azimuthal symmetry, that under $\boldsymbol{E}\times\boldsymbol{B}$ fields is three-variable. Its calculation requires a huge memory capacity and the computational load is heavy. It was recent that computers became capable of such calculations. A prototype PM code for the EVDF under $\boldsymbol{E}\times\boldsymbol{B}$ fields was composed with customized propagators (Green's functions) to deal with intercellular electron transfers due to acceleration and collisional scattering. This report presents results of benchmark calculations.

2. Field model and PM configuration

In three-variable velocity space (υ, θ, ϕ) related to velocity $\boldsymbol{\upsilon} = (\upsilon_x, \upsilon_y, \upsilon_z)$ as $\upsilon_x = \upsilon\sin\theta\cos\phi$, $\upsilon_y = \upsilon\cos\theta$ and $\upsilon_z = \upsilon\sin\theta\sin\phi$ under $\boldsymbol{E} = (0, 0, -E)$ $(E > 0)$ and $\boldsymbol{B} = (0, B, 0)$, the (i, j, k)th cell was defined as the region of $\upsilon_{i-1} \le \upsilon < \upsilon_i$, $(j-1)\Delta\theta \le \theta < j\Delta\theta$ and $(k-1)\Delta\phi \le \phi < k\Delta\phi$. Here, $\upsilon_i = \upsilon_{1\mathrm{eV}}(i\Delta\varepsilon/\varepsilon_{1\mathrm{eV}})^{1/2}$, $\upsilon_{1\mathrm{eV}}$ is the electron speed associated with 1 eV and $\varepsilon_{1\mathrm{eV}} = 1$ eV. Desirable resolution depends on gas medium, E/N and B/N, where N is the gas molecule number density. The present PM adopted $\Delta\varepsilon = 0.01$ eV for 0–100 eV, $\Delta\theta = \pi/90$ and $\Delta\phi = 2\pi/360$ for SF_6 at $N = 10^{22}$ m^{-3}.

Electron acceleration $\boldsymbol{a} = (a_x, a_y, a_z) = \mathrm{d}\boldsymbol{\upsilon}/\mathrm{d}t = (\mathrm{d}\upsilon_x/\mathrm{d}t, \mathrm{d}\upsilon_y/\mathrm{d}t, \mathrm{d}\upsilon_z/\mathrm{d}t)$ by the $\boldsymbol{E}\times\boldsymbol{B}$ fields is dependent on $\boldsymbol{\upsilon}$ as $a_x = (e/m)\upsilon_z B$, $a_y = 0$ and $a_z = (e/m)(E - \upsilon_x B)$, where e and m are the electronic charge and mass, respectively. $\boldsymbol{a}$ is rotational around an axis in velocity space. The intercellular electron transfer due to $\boldsymbol{a}$ was evaluated as the integral of $\boldsymbol{\Gamma}\cdot\boldsymbol{n}$ over a cell surface, through which electrons move to the downstream neighbour cells. Here, $\boldsymbol{\Gamma}$ and $\boldsymbol{n}$ are the electron flux and the normal vector at the cell surface, respectively.

The collision propagator represents discontinuous changes of υ and θ at scatterings. Isotropic scattering was assumed for the collision propagator.

Table 1. Components $(\langle\upsilon_z\rangle, \langle\upsilon_x\rangle)$ $(10^4$ m s$^{-1})$ of the average electron velocity under the Hall deflection: υ_z, component of the $-\boldsymbol{E}$ direction; and υ_x, component of the $\boldsymbol{E}\times\boldsymbol{B}$ direction.

	Method	100 Hx	200 Hx	500 Hx
100 Td	MC	(6.39, 0.44)	(6.30, 0.86)	(5.79, 1.99)
	PM	(6.37, 0.43)	(6.29, 0.86)	(5.76, 1.96)
200 Td	MC	(11.44, 0.68)	(11.31, 1.36)	(10.59, 3.21)
	PM	(11.42, 0.69)	(11.31, 1.37)	(10.57, 3.22)
500 Td	MC	(23.89, 1.19)	(23.71, 2.36)	(22.68, 5.74)
	PM	(23.90, 1.20)	(23.74, 2.38)	(22.68, 5.74)

The EVDF was relaxed from a Maxwellian by applying the propagators to the EVDF using a numerical scheme like the Gauss–Seidel method. Components $\langle\upsilon_z\rangle$ and $\langle\upsilon_x\rangle$ of the average electron velocity under the Hall deflection were derived and compared with those obtained by MC simulations.

3. Results

Table 1 shows MC and PM results of $\langle\upsilon_z\rangle$ and $\langle\upsilon_x\rangle$ in SF_6 at E/N = 100–500 Td (1 Td = 10^{-21} Vm^2) and B/N = 100–500 Hx (1 Hx = 10^{-27} Tm^3). They agree with each other within discrepancies of a few percent.

4. Summary

A basic calculation scheme of the PM for EVDFs under $\boldsymbol{E}\times\boldsymbol{B}$ fields was established. Improvement of the precision via adjustment of resolution and further extension of the PM to real-space electron transport parameters are considered as succeeding work.

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Reference

[1] H. Sugawara, Plasma Sources Sci. Technol. (2017) (at press) doi: 10.1088/1361-6595/aa5d7f .